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⁂ *This Key applies only to the* SECOND *Edition of the Author's Elementary Treatise on Logarithms and Trigonometry.*

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KEY

TO THE

ELEMENTARY MATHEMATICS.

PART THIRD.

LOGARITHMS

AND

PLANE TRIGONOMETRY.

LOGARITHMS

AND

PLANE TRIGONOMETRY.

CHAPTER I.

DEFINITIONS AND EXPLANATIONS OF LOGARITHMS.

No answers or solutions are required in this chapter.

CHAPTER II.

ON THE USE OF LOGARITHMS.

PROBLEM I.

The answers to the Exercise will be seen at once in Table I.

PROBLEM II.

ANSWERS TO UNRESOLVED EXERCISES.

0, 1, 3, 1, 2, 2, 0, 0, and 5.

PROBLEM III.

ANSWERS TO UNRESOLVED EXERCISES.

-1, -1, -2, -4, and -3.

Problem IV.

ANSWERS TO UNRESOLVED EXERCISES.

A. 2·750508; 2·850033; and 2·740363.
B. 2·562293; 2·313867; and 2·954243.
C. 1·390935; and 0·588832.
D. 3·931458; and 4·863323.
E. 1·431364; 0·591065; and 0·903090.
F. $\bar{1}\cdot096910$; $\bar{1}\cdot682145$; $\bar{3}\cdot732394$; and $\bar{4}\cdot954243$.

Problem V.

ANSWERS TO UNRESOLVED EXERCISES.

A. 3·660865; 3·661434; 1·941511; 1·941611; 0·525951; and 2·970068.

B. 4·692053; 6·894094; 5·580697; and 7·857332.

C. 4·412629; 3·412629; 2·412629; 1·412629; and 0·412629.

D. $\bar{1}\cdot930592$; $\bar{2}\cdot860218$; and $\bar{4}\cdot860218$.

Problem VI.

Note. The last paragraph of the rule rests on the assumption that the difference of the logarithm corresponding to half a unit in the fourth figure of the natural number, is half the difference corresponding to a whole unit, which it is *nearly*.

ANSWERS TO UNRESOLVED EXERCISES.

A. 4·424555; 3·925621; 2·216166; and 0·993480.
B. $\bar{1}\cdot642168$; $\bar{2}\cdot485011$; 1·993172; and $\bar{3}\cdot803730$.
C. 4·392433; 1·991780; and $\bar{4}\cdot950681$.

Problem VII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 0·338818; $\bar{1}\cdot823909$; $\bar{1}\cdot823909$; $\bar{3}\cdot823909$; and $\bar{5}\cdot000000$.

B. $\bar{1}\cdot698970$; $\bar{1}\cdot875061$; $\bar{1}\cdot903090$; and $\bar{1}\cdot574031$.

Problem VIII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 1·261263 ; 0·960233 ; 1·682596 ; and 1·475308.
B. 0·823909 ; 2·562491 ; and 1·157264.

Problem IX.

ANSWERS TO UNRESOLVED EXERCISES.

A. 100 ; 1660 ; 677,900 ; 39·55 ; and 0·1334.
B. 66,090 ; 186·8 ; and 0·3982.
C. 23·50 + ; 296·8 – ; 6·886 – ; and ·02819 +.
D. 4·7705 ; 6,020,500 ; 0·15005 + ; and ·092495 –.

Problem X.

ANSWERS TO UNRESOLVED EXERCISES.

A. 852·4 –, and 1771 –.
B. 15·41 –, and 5065 +.
C. 419·5 +.
D. 7151 –.
E. 3443 +.
F. 23,37(0,000 –).*
G. ·08711 –, and ·00009529 –.
H. ·003283 +, and 0·9604 –.
K. 9·417 +, and 13·59 –.
L. 0·3616 –, and 33·67 –.
M. ·07023 –.
N. 0·9104 –.

Problem XI.

ANSWERS TO UNRESOLVED EXERCISES.

A. 36, and 45.
B. 2·1, and 121·4 +.
C. 0·3600, and ·02078 –.
D. 1·707 +, and 1·318 –.
E. 72·33 +, and 13·86 + .
F. 1261 +, and 5·476 + .
G. 0·1825 –, ·01382 + .
H. 0·4273 –, ·00007056 +.

---

* In such an expression, the sign – does not apply merely to the last cipher, but is intended to show that the fourth figure is too great.

Problem XII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 1533.
B. 45·55 –.
C. 253·2 –.
D. 0·2331 –.
E. 11·66 +.
F. 2·851 +.
G. 1·879 +.
H. 1·284 +.
K. ·08877 –.
L. ·05756 +.
M. ·009581 +.
N. ·0005469 +.

Problem XIII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 625 ; 1467 –; and 30·21.
B. 3375 ; 254·0 –; and 6332 –.
C. 147,8(00 +) ; and 7,762,(000 +).
D. ·0003204 + ; 0·7270 + ; and ·0000790(1 +).
E. ·006476 + ; ·02279 –; and ·0007266 –.
F. ·004096 ; 446·7 + ; and ·000,000,009,766 –.

Problem XIV.

ANSWERS TO UNRESOLVED EXERCISES.

A. 290 ; 9·4 ; 534·0 + ; and 3·162 +.
B. 17 ; 150 ; 0·500 + ; and 8·434 +.
C. ·02345 + ; ·09 ; 0·8112 – ; and 0·6666 +.
D. 0·21 ; 0·7937 + ; 0·4098 – ; and ·08363 +.
E. 8 ; 3·242 + ; 2·154 + ; 0·1326 – ; 0·2854 –.

CHAPTER III.

ON THE APPLICATIONS OF LOGARITHMS.

ANSWERS TO UNRESOLVED EXERCISES.

A. 509·2 +.
B. 189·4 +.
C. 14,390 + st.
D. £11·34 –.
E. £1·910 –.
F. £4670·97 +.
G. £71·06 +.
H. £163,(800,000).
K. £79·8 –.
L. 8·714 +.
M. 4·9025 +.
N. 2·790 ±.
O. 129·9 +.
P. 2·552 –.
Q. 2·665 +.
R. 27·30 +.
S. 1882 –.
T. 2·956 – oz.

OPERATION OF EXERCISE 1.

379..................2·578639
1728................3·237544

277·3...............2·442950

2362 –..............3·373233

EXERCISE A.

556 × 1612 ÷ 1760.

EXERCISE 3.

$$\left.\begin{matrix}702\text{ q.} : 1464\text{ q.}\\ 5\text{ m.} : \ \ 14\text{ m.}\end{matrix}\right\} :: 939\text{ men.}$$

939..................2·972666
1464................3·165541
14....................1·146128

7·284335

702..................2·846337
5......................0·698970

3·545307

5483 +.............3·739028

EXERCISE C.

$$\left.\begin{matrix}1354 : 1706\\ 127 : \ \ 119\end{matrix}\right\} :: 12190.$$

EXERCISE 4.

386 : 3½ × 1·261 :: 19740.

19740..............4·295347
3·5...................0·544068
1·261...............0·100715

386..................2·586587

225·7 +............2·353543.

EXERCISE 5.

$$\left.\begin{matrix}100 : 1578\\ 365 : \ \ 139\end{matrix}\right\} :: 4\tfrac{1}{4}.$$

4·25.................0·628389
1578................3·198107
139..................2·143015

100..................2·000000
365..................2·562293

25·54 –............1·407218.

EXERCISES D, E, AND F.

457·4 × 4·25 × 7 ÷ 1200.
64·54 × 3·75 × 288 ÷ 36500.
4651 × 38 × 4·125 ÷ 36500.

EXERCISE 8.

Amount ≑ $320·5 \times 1·05^4$.

1·05...............0·021189
4

0·08475(6)
320·5.............2·505828

389·6 –..........2·59058(4).

EXERCISE 9.

Amount ≑ $857 \times 1·0375^6$.

1·0375.............0·015988
6

0·095928
857..................2·932981

1069 –.............3·028909.

EXERCISES G. AND H.

G. Amount ≑ $10 \times 1·04^{50}$.
H. Amount in Pounds ≑ $1 \times 1·05^{500} \div 240$.

EXERCISE 10.

1·035	0·014940
	9
	0·13446(0)
15·500	1·190332
21·125 –	1·32479(2).
5·625 –.	

EXERCISE K.

$$\text{Amount} \doteq 383 \times 1{\cdot}035^{55}.$$

EXERCISE 21.

31·62 ch.	1·499962
7·37	0·867467
7·44	0·871573
16·81	1·225568
	2)4·464570
170·7 +	2·232285.

EXERCISE 22.

68·5 in	1.835691
33·5	1·525045
17·5	1·243038
17·5	1.243038
	2)5·846812
	2·923406
144	2·158362
5·822 –	0·765044.

EXERCISE L.

$$\text{Area} \doteq \sqrt{(6{\cdot}75 \times 2 \times 2{\cdot}25 \times 2{\cdot}5)}.$$

EXERCISE M.

$$\text{Area in square chains}$$
$$\doteq \sqrt{(16{\cdot}315 \times 3{\cdot}755 \times 5{\cdot}825 \times 6{\cdot}735)}.$$

EXERCISE 23.

$$\triangle ABC \doteq \sqrt{(9{\cdot}635 \times 3{\cdot}355 \times 4{\cdot}995 \times 1{\cdot}285)}$$
$$= 14{\cdot}405 - \text{sq. chains.}$$
$$\triangle ADC \doteq \sqrt{(10{\cdot}125 \times 5{\cdot}555 \times 2{\cdot}795 \times 1{\cdot}775)}$$
$$= 16{\cdot}705 - \text{sq. chains.}$$

EXERCISE N.

$$\triangle ABD \doteq \sqrt{(9{\cdot}58 \times 2{\cdot}13 \times 2{\cdot}85 \times 4{\cdot}60)}$$
$$= 16{\cdot}36 - \text{sq. chains.}$$
$$\triangle BCD \doteq \sqrt{(7{\cdot}83 \times 2{\cdot}01 \times 2{\cdot}97 \times 2{\cdot}85)}$$
$$= 11{\cdot}54 + \text{sq. chains.}$$

EXERCISE 24.

Area in acres

$$\doteq 89^2 \times \frac{\Pi}{4} \div 4840.$$

89	1·949390
89	1·949390
0·7854 −	$\bar{1}$·895090
4840	3·684845
1·285 +	0·109025.

EXERCISE 25.

$$\text{Area} \doteq 32{\cdot}25^2 \times \frac{1}{4\Pi}.$$

32·25	1·508530
32·25	1·508530
·07958 −	$\bar{2}$·900790
82·77 −	1·917850.

EXERCISES O, P, Q, R.

O. Diam. $\doteq 408{\cdot}2 \div (3{\cdot}1416 -)$.
P. Area $\doteq 2{\cdot}85^2 \times (3{\cdot}1416 -)$, or
Area $\doteq 5{\cdot}7^2 \times ({\cdot}7854 -)$.
Q. Area $\doteq 549^2 \times {\cdot}07958 \div 9$.
R. Area $\doteq 7{\cdot}38 \times 4{\cdot}71 \times (3{\cdot}1416 -)$.

EXERCISE 26.

$$\text{Cont.} \doteq 42 \times 15^2 \times \frac{\Pi}{4} \div 1728.$$

42	1·623249
15	1·176091
15	1·176091
0·7854 −	$\bar{1}$·895090
1728	3·237544
4·295 +	0·632977.

EXERCISE 27.

Content $\doteq$
$$4{\cdot}18^2 \times 6{\cdot}99 \times {\cdot}43301 \div 3.$$

4·18	0·621176
4·18	0·621176
6·99	0·844477
0·4330	$\bar{1}$·636488
3)52·88 +	1·723317.
17·63 −.	

EXERCISE S.

C. $\doteq 4{\cdot}18^2 \times {\cdot}43301 \times 98 \div 3$.

EXERCISE 28.

Content $\doteq 5{\cdot}88^2 \times 7{\cdot}74 \times \frac{\Pi}{6}$.

5·88..................0·769377
5·88..................0·769377
7·74..................0·888741
0·5236 – ....$\bar{1}$·718999

140·1 + ...2·146494.

EXERCISE 29.

As $3{\cdot}5^3 : 6{\cdot}7^3 :: 6 :$ ——.

6·7..................0·826075
3·5..................0·544068

0·282007
3

0·846021
6·0 ....0·778151

42·09 – ....1·624172.

EXERCISE T.

Weight $\doteq 1{\cdot}75^3 \times {\cdot}5236 \div 1728 \times 1820$.

CHAPTER IV.

OF LOGARITHMIC SCALES.

No answers or solutions are required in this chapter.

CHAPTER V.

DEFINITIONS AND EXPLANATIONS OF PLANE TRIGONOMETRY.

ANSWERS TO UNRESOLVED EXERCISES.

D. The sum is equal to the radius.

E. Sine $90° \doteq 10$; chord $60° \doteq 10$; sine $30° \doteq 5$; cos $60° \doteq 5$; and versin $60° \doteq 5$.

F. Tan $45° \doteq 1$; cot $45° \doteq 1$; sec $60° \doteq 2$; cos $0° \doteq 1$; and tan $90° \doteq$ Infinity.

G. Sine $60° \doteq 0{\cdot}86603 -$; tan $60° \doteq 1{\cdot}73205 +$; and sec $45° \doteq 1{\cdot}4142 +$.

H. Sine 30° ≐ 50; sine 150° ≐ 50; sine 60° ≐ 86·6 +; and sine 120° ≐ 86·6 +.

K. Because no sine is greater than the radius, and no secant less than the radius.

L. Because the tangent of an angle of less than 45° is smaller than the radius, while that of an angle of more than 45° is greater than the radius.

CHAPTER VI.

ON THE USE OF THE TABLES OF SINES, TANGENTS, AND SECANTS.

Problem I.

ANSWERS TO UNRESOLVED EXERCISES.

A. 9·609313; 9·967166; 9·764308; 9·997797.
B. 9·799157, and 9·875571.
C. 9·977209, and 10·193140.

Problem II.

ANSWERS TO UNRESOLVED EXERCISES.

A. 9·588590; 10·300684; 10·104855; 9·383168.
B. 9·100559; 10·274478; and 10·100970.
C. 9·982090; and 9·941841.

Problem V.

ANSWERS TO UNRESOLVED EXERCISE.

A. 9·986331; 9·906852; and 9·415347.

Problem VII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 19° 21′; 68° 38′; and 74° 15′.
B. 11° 6′; 74° 17′ −; and 85° 46′ +.

C. 70° 17½′; and 35° 40½′.
D. 8° 11′; 17° 13′+; and 68° 45′+.

CHAPTER VII.

OF RIGHT-ANGLED TRIANGLES.

PROBLEM I.

ANSWERS TO UNRESOLVED EXERCISES.

A. 71¼°.
B. 5° 46½′.
C. 61° 15′ 5″.

PROBLEM II.

ANSWERS TO UNRESOLVED EXERCISES.

A. 3119+.
B. 1·711+.
C. 4·831+.
D. 806·3−.
E. ·02762−, and ·02893+.

PROBLEM III.

ANSWERS TO UNRESOLVED EXERCISES.

A. 960·8−.
B. 36·09+.
C. 39·74−.
D. 1808+

PROBLEM IV.

ANSWERS TO UNRESOLVED EXERCISES.

A. 45·36−.
B. 1 ft. 4½ in.+
C. 340·5+.
D. 0·2932+.

PROBLEM V.

ANSWERS TO UNRESOLVED EXERCISES.

A. 31° 41′−.
B. 40° 19′+, and 49° 41′−.

C. 46° 40′+, and 43° 20′−, if resolved by decimals, but more correctly 46° 39½′−, and 43° 20½′+, by vulgar fractions.

PROBLEM VI.

ANSWERS TO UNRESOLVED EXERCISES.

A. 56° 19′ −, and 30° 41′ +.
B. 53° 8′ −, and 36° 52′ +.
C. 2° 3′ −.
D. 63° 26′ +, and 26° 34′ −.

PROBLEM VII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 5 ft. 7·4 in. − | B. 7500. | C. 0·1290 −.

The solutions by Rules 1 and 2 need not be given here, since they are short and simple. The following are the processes *by Rule* 3.

EXERCISE 1.

Log sin P	9·945643.
Nearest on Table	9·945666.
Corresponding cosine, or sin B	9·672558.

EXERCISE 2.

Log sin B	9·903125.
Nearest on Table	9·903108.
Cor. cosine, or sin P	9·778119.

EXERCISE A.

Log sin B	9·874706.
Nearest on Table	9·874680.
Cor. cosine, or sin P	9·820979.

EXERCISE B.

Log sin P	9·585027.
Nearest on Table	9·584968.
Cor. cosine, or sin B	9·965248.

EXERCISE 3.

Log sin B..............................9·945594.
Nearest on Table....................9·945598.
Cor. cosine, or sin P................9·672795.

EXERCISE 4.

Log sin B..............................9·341580.
Nearest on Table....................9·341558.
Cor. cosine, or sin P................9·989271.

EXERCISE C.

Log sin B............................9·305296.
Nearest on Table..................9·305207.
Cor. cosine, or sin P..............9·990960.

PROBLEM VIII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 12·7 −. | B. 131 +. | C. ·0870 −.

SOLUTIONS BY RULE 2.

EXERCISE 1.

Log tan P ≐ 10·273002. Nearest cor. sec. ≐ 10·327442.

EXERCISE A.

Log tan P ≐ 9·026410. Nearest cor. sec. ≐ 10·002439.

EXERCISE B.

Tan P ≐ 10 694606. Nearest cor. secant ≐ 10·703461.

EXERCISE C.

Tan P $\doteq$ 9·682025. Nearest cor. secant $\doteq$ 10·045177.

EXERCISE 2.

Log tan B $\doteq$ 9·875047. Nearest cor. sec $\doteq$ 10·096892.

EXERCISE 3.

Log tan B $\doteq$ 10·272976. Nearest cor. sec $\doteq$ 10·327442.

EXERCISE 4.

Log tan B $\doteq$ 9·352339. Nearest cor. sec $\doteq$ 10·010729.

PROMISCUOUS EXERCISES IN RIGHT-ANGLED TRIANGLES.

ANSWERS TO UNRESOLVED EXERCISES.

A. 20. B. 14·92 −. C. 130·2 −, and 318·4 +.
D. 1059 +. E. 28° 58′ −, or, more correctly, 28° 57′ +.
F. 115·5 −. G. Alt., 168·0(+). Angles, each 60° 31′ −.
H. 53° 24′ −. K. 30·78 −, and 32·36 +.
L. The one is twice the other.
M. 0·9511 −. N. 10·85 −.

EXERCISE 2.

This may be done by any of the three Rules of Problem VII. The processes by Rules 1 and 2 are simple. *By Rule* 3 the following are the steps :—

Log sin B $\doteq$ 9·341580. Nearest cor. cos. $\doteq$ 9·778119.

EXERCISE B.

As rad : sin 22° 30′ :: 19·49 : half the base, or
As radius : sin 22° 30′ :: 38·98 : base.

EXERCISE C.

As radius : sin 22° 14′ :: 344 : shorter side.
As radius : cos 22° 14′ :: 344 : longer side.

EXERCISE 3.

The smaller acute angle must be 1 fifth of 90°, or 18°; the other being 4 fifths of 90°, or 72°. Then,

As rad : sin 18° :: 100 : *b*, and
As rad : sin 72° :: 100 : *p*.

EXERCISE 4.

The angles depend upon the ratio of the legs, and not on their actual magnitude: we may therefore fix them at any two numbers we please in the ratio of 4 to 1. We therefore say—

As 4 : 1 :: radius : tan P, or cot B.

EXERCISE 5.

A perpendicular, drawn from the vertex to the base, divides the isosceles triangle into two right-angled triangles, in each of which the hypothenuse is 8 and the base 1, or two numbers in the same proportion. Therefore,

As 8 : 1 :: radius : sin B, or cos P.
$\therefore \angle B \doteq 7^\circ\ 11' -$, and $\angle P = 82^\circ\ 49' +$.

The latter of these is one of the equal angles at the base of the isosceles triangle, and the former is half the angle at the vertex.

EXERCISE D.

As radius : sin 45° :: 749 : side of square.

EXERCISE 6.

Draw the diameter, the chord, and the chord of the supplement. We thus form a triangle, of which the angle contained between the two chords is a right angle, being the angle in a semicircle, and the angle opposite the given chord is an angle of 15°, since the angle at the circumference is half the angle at the centre. (Euc. III. 31 and 20). Therefore,

As radius : sin 15° :: 1000 : 258·8 + .

EXERCISE 7.

As radius : sin 60° :: ·08 : altitude.

EXERCISE 8.

As has just been shown, in Exercise 6, the diameter and the two chords form a right-angled triangle. Then, from Problem VIII.,

By Rule 1, $h \doteq \sqrt{(2{\cdot}148^2 + 0{\cdot}895^2)}$.

By Rule 2,

Log tan B ≐ 9·619789. Nearest cor. sec. ≐ 10·034752.

EXERCISE 9.

The sine and the cosine are two legs of a right-angled triangle, of which the radius is the hypothenuse. Therefore, by Problem VIII., Rule 2, calling the given sine p,

Log tan P ≐ 9·995093. Nearest cor. sec. ≐ 10·148128.

EXERCISE 10.

By Pr. VII., Rule 2,

$h + p = 124{\cdot}6$............Log. 2.095518
$h - p = 4{\cdot}983$..................0·697491
2)2·793009
Answer, 24·92 – ....1·396504.

By Rule 3,

Log sin P ≐ 9·965260 Nearest cor. cos. ≐ 9·584968.

EXERCISE H.

(745 – 32) ÷ 2 ≑ 356·5.
As 356·5 : 480 :: radius : tan P.

EXERCISE K.

As radius : tan (72° = $\frac{4}{5}$ of 90°) :: 10 : longer leg.
As radius : sec 72° :: 10 : hypothenuse.

EXERCISES L, M, and N.

L. As 2 : 1 :: radius : sin B, or cos P.
M. As radius : cos 18° :: 1 : rad. insc. circ.
N. As radius : sin (25° 43′ – = $\frac{1}{14}$ of 360°) :: 25 : side.

CHAPTER VIII.

OF OBLIQUE-ANGLED TRIANGLES.

PROBLEM I.

ANSWERS TO UNRESOLVED EXERCISES.

A. 86$\frac{3}{4}$^{c}.
B. 31° 43′.
C. 57° 51′ 52″.
D. 65° 11$\frac{1}{4}$′.
E. 102° 45′ –.

PROBLEM II.

ANSWERS TO UNRESOLVED EXERCISES.

A. 958·6 +.
B. 9·901 +, and 8·8945 –.
C. 0·9119 –, and 0·4825 –.
D. 7 yds. 0·43 + ft.
E. 4 yds. 1·04 – ft.

EXERCISE 2.

As sin 95° 5′ : sin 38° 24′ :: 7·832 : 4·884 +.
Sin 95° 5′ ≑ sin 84° 55′.

EXERCISE 4.

As sin 48° 27′ : sin 63° 55′ :: 5·75 : 6·901 −.
As sin 48° 27′ : sin 67° 38′ :: 5·75 : 7·105 +.

EXERCISE 5.

As sin 56° 26′ : sin 61° 47′ :: 473 in. : 500 + in.

EXERCISE 6.

As sin 64° 31′ : sin 91° 5′ :: 784·4 : 869 −.

PROBLEM III.

ANSWERS TO UNRESOLVED EXERCISES.

A. 58° 58′ +.
B. 77° 40′ +.
C. 123° 30′ −.
D. 62° 17′ −.
E. 67° 24′ +, or 112° 36′ −.
F. No answer.

EXERCISE F.

If we say—As 79 : 86 :: sin 73° : sine of required angle; and if we compute this by logarithms, we find the logarithm of the required sine 10·017467, which is impossible, since no sine is greater than the radius. In fact the data are inconsistent, and it is not possible to construct a triangle of the dimensions given.

PROBLEM IV.

ANSWERS TO UNRESOLVED EXERCISES.

A. 0·305 +.
B. 4·55 +.
C. 0·146 +.
D. 373 −.
E. 9·82 +, or 2·14 −.

EXERCISE 1.

We first find ∠C = 58° 12′.
Hence, ∠B ≐ 57° 16′.
As sin 64° 32′ : sin 57° 16′ :: 409 : 381·1 − = *b*.

EXERCISE 2.

$\angle B \doteq 134^\circ\ 40'$. $\therefore \angle C \doteq 21^\circ\ 35'$. Hence,
As $\sin 23^\circ\ 45' : \sin 21^\circ\ 35' :: a = 4{\cdot}861 : c = 4{\cdot}44 -$.

EXERCISE 3.

$\angle C \doteq 38^\circ\ 23' +$. $\therefore \angle B \doteq 46^\circ\ 22' -$. Hence,
As $\sin 95^\circ\ 15' : \sin 46^\circ\ 22' :: 7{\cdot}832 : b = 5{\cdot}692 +$.

EXERCISE 4.

$\angle C \doteq 47^\circ\ 15' -$ or $47^\circ\ 14\frac{1}{2}' +$.
$\therefore \angle B \doteq 54^\circ\ 31' +$ or $54^\circ\ 31\frac{1}{2}' -$.
$\therefore$ As $\sin 78^\circ\ 14' : \sin 54^\circ\ 31'$ or $54^\circ\ 31\frac{1}{2}'$
$:: 4 : b = 3{\cdot}327 +$.

EXERCISE 5.

$\angle C \doteq 63^\circ\ 55'$ or $116^\circ\ 5'$.
$\angle B \doteq 67^\circ\ 38'$ or $15^\circ\ 28'$. Hence,
As $\sin 48^\circ\ 27' : \sin 67^\circ\ 38' :: {\cdot}0575 : b = {\cdot}0711 -$, or
As $\sin 48^\circ\ 27' : \sin 15^\circ\ 28' :: {\cdot}0575 : b = {\cdot}0205 -$.

PROBLEM V.

ANSWERS TO UNRESOLVED EXERCISES.

A. $66^\circ\ 43' -$, and $47^\circ\ 17' +$.
B. $97^\circ\ 3' -$, and $64^\circ\ 27' +$.
C. $45^\circ\ 15' -$, and $38^\circ\ 25' +$.

EXERCISE 1.

As $790 : 28 :: \tan 60^\circ\ 54' : \tan 3^\circ\ 39' -$.

EXERCISE 2.

As $18{\cdot}6 : 0{\cdot}84 :: \tan 78^\circ\ 7' : \tan 12^\circ\ 7' -$.

EXERCISE 3.

As 1·5 : 0·186 :: tan 41° 17′ : tan 6° 13′ −.

EXERCISE B.

As 18·375 : 0·875 :: tan 80° 45′ : tan 16° 18′ −.

Or, if we multiply the lineal dimensions by 8, it will not affect the angles. We may therefore say—

As 147 : 7 :: tan 80° 45′ : tan 16° 18′ −.

EXERCISE C.

Multiplying the lineal dimensions by 3, we say—

As 15 : 1 :: tan 41° 50′ : tan 3° 25′ −.

EXERCISE 4.

As 14005 : 202·2 :: tan 65° 46½′ : tan 1° 50′ +.

PROBLEM VI.

ANSWERS TO UNRESOLVED EXERCISES.

A. 4·97 +. | B. 3·08 −. | C. 3·73 +.

EXERCISE 1.

As 15·8 : 0·56 :: tan 60° 54′ : tan 3° 38½′ +.
$\therefore \angle A \doteq 64° 32\frac{1}{2}' +$, and $\angle C = 57° 15\frac{1}{2}' -$. Then,
As sin 64° 32½′ : sin 58° 12′ :: 8·18 : 7·700 −.

Proof.—As sin 57° 15½′ : sin 58° 12′ :: 7·62 : 7·700 −.

EXERCISE 2.

As 105·7 : 8·9 :: tan 66° 14′ : tan 10° 49½′ −.
$\angle A \doteq 77° 3\frac{1}{2}' -$. $\angle B \doteq 55° 24\frac{1}{2}' +$.
As sin 77° 3½′ : sin 47° 32′ :: 57·3 : 43·37 +.

Proof.—As sin 55° 24½′ : sin 47° 32′ :: 48·4 : 43·37 +.

EXERCISE 3.

As 0·17 : ·07 :: tan 45° : tan 22° 23′ −.
∠A ≐ 67° 23′ −. ∠C ≐ 22° 37′ +.
As sin 67° 23′ : sin 90° :: 0·12 : 0·1300(−).

Proof.—As sin 22° 37′ : sin 90° :: 0·05 : 0·1300(+).

EXERCISE 4.

Having found the angles at the base, as in Exercise 4 of Problem v., to be 67° 37′ − and 63° 56′ +, we say—

As sin 67° 37′ : sin 48° 27′ :: 71·04 : 57·50 −.

Proof.—As sin 63° 56′ : sin 48° 27′ :: 69·01 : 57·49 +.

PROBLEM VII.

ANSWERS TO UNRESOLVED EXERCISES.

A. 57° 15′ +, 58° 12′ +, and 64° 33′ −.
B. 47° 32′ +, 55° 24′ +, and 77° 4′ −.

EXERCISE 1.

As 735 : 945 :: 245 : 315 = *d*.
CD ≐ 525. AD ≐ 210.

EXERCISE 2.

Since it is the proportion of the sides, and not their actual values, which determines the angles, we may as well remove the decimal point, at once, three places to the right, in each of the given numbers, making all the figures integers—viz., 532, 259, and 371. Then,

As 532 : 630 :: 112 : 132·6 + = *d*.
CD ≐ 332·3 +. AD ≐ 199·7 −.

EXERCISE 3.

The three sides, reduced to inches, are 776·5, 547·5, and 435. Then,

$$\text{As } 776{\cdot}5 : 982{\cdot}5 :: 112{\cdot}5 : 142{\cdot}3 + = d.$$
$$\text{CD} \doteq 459{\cdot}4. \quad \text{AD} \doteq 317{\cdot}1.$$

EXERCISE A.

$$\text{As } 1636 : 3064 :: 16 : 29{\cdot}97 - = d.$$
$$\text{CD} \doteq 833{\cdot}0 -. \quad \text{AD} \doteq 803{\cdot}0 +.$$

EXERCISE B.

$$\text{As } 11{\cdot}46 : 18{\cdot}35 :: 1{\cdot}006 : 1{\cdot}611 -.$$
$$\text{CD} \doteq 6{\cdot}535(5). \quad \text{AD} \doteq 4{\cdot}924(5).$$

CHAPTER IX.

ON THE APPLICATIONS OF PLANE TRIGONOMETRY.

ANSWERS TO UNRESOLVED EXERCISES.

A. 35·21 feet.
B. 34′ +.
C. 1 in 38, or 138 feet per mile, nearly.
D. 268·3 – feet.
E. S 41° 59′ + W.
F. 9° 1′ –.
G. Departure = 4·59 +.
Dif. of Lat. = 11·09 –.
H. Distance = 87·68 + m.
Dif. of Lat. = 17·11 – m.
I. S 65° 5′ + W, or
SW by W ¾ W, nearly.
J. 59·14 +.
K. 254·6 – and 239·2 + ft.
L. 30° 7′ –.
M. 59·8 + feet.
N. 653 + and 691 – links.
O. 96·38 – feet.
P. 164·9 – feet.
Q. 93·68 – feet.
R. 1173 + and 1126 + ft.
S. 229·9 + yards.
T. A, 636·4 –,
B, 882·7 – yards.
U. 102·0 + feet.
V. 6536 – feet.
W. Distance 722 –,
Height 269 + feet.
X. 196·8 – feet.

Y. 211·5 – and 219·9 + ft.
Z. = 148·2 – feet.
AA. 58·23 – feet.
BB. 126 – feet.
CC. ER = 4077(0 –),
PR = 2053 + feet.

DD. Distance, 2·105 – miles.
Height, 433 + feet.
EE. 7285 + feet.
FF. 1002 + feet.
GG. 1° 49½′.
HH. ·077 +, 2·054 –, and 4·077 +.

EXERCISE 2.

Log tan of required angle ≐ 8·070581.
This is the tangent of 0° 40′ nearly.

EXERCISE 3.

As radius : tan 89° :: 1 : 57·29, and
As radius : tan 1° :: 5280 : 92·16 +.

EXERCISE 4.

As radius : cos 1° 43′ :: 854·9 : 854·5 +.

EXERCISE 5.

The three towns are situate at the angular points of a right-angled triangle, A being at the right angle.

As 19 : 17 :: radius : tan 41° 49′ +.

EXERCISE 6.

As 20·6 : 3·7 :: radius : tan 10° 11′ –.

EXERCISE 7.

As radius : sin 33° 45′ :: 81 : 45·00 + = Departure.
As radius : cos 33° 45′ :: 81 : 67·35 – = Dif. Lat.

EXERCISE 8.

$$\text{SSW} \doteq \text{S } 22^\circ\ 30'\ \text{W}.$$
$$90^\circ - 22^\circ\ 30' \doteq 67^\circ\ 30'.$$
$$\text{As radius} : \sec 67^\circ\ 30' :: 198 : 517{\cdot}4 -.$$
$$\text{As radius} : \tan 67^\circ\ 30' :: 198 : 478{\cdot}0 +.$$

EXERCISE 9.

$$\text{As } 255 : 157{\cdot}5 :: \text{radius} : \cos 51^\circ\ 51' +.$$

EXERCISE 13.

$$\text{As radius} : \sec 63^\circ\ 42\tfrac{1}{2}' :: 35 \text{ yds.} : 79{\cdot}02 -.$$
$$\text{As radius} : \tan 63^\circ\ 42\tfrac{1}{2}' :: 35 \text{ yds.} : 70{\cdot}84 +.$$

EXERCISE 14.

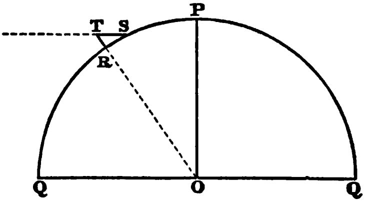
$$\text{As } 280 \text{ inches} : 185 \text{ inches} :: \text{radius} : \tan 33^\circ\ 27' +.$$

EXERCISE L.

$$\text{As } 25 \text{ feet} : 14{\cdot}5 \text{ feet} :: \text{radius} : \tan 30^\circ\ 7' -.$$

EXERCISE 15.

Let QQ be the Earth's Equator; P, the Pole; TR, the tree; and RS the shadow on the ground, which, being of so small extent, may be regarded as a straight line. TS, the direction of a ray of light at T, will be parallel to QQ, the distance of the sun being practically the same as infinite. Then TRS is a right angle, and



$$\angle \text{STR} = \angle \text{ROQ} \doteq 55\tfrac{1}{2}^\circ,$$
$$\angle \text{TSR} \ldots\ldots\ldots\ldots \doteq 34\tfrac{1}{2}^\circ.$$
$$\text{As radius} : \tan 34\tfrac{1}{2}^\circ :: \text{SR} : \text{TR} :: 96 : 65{\cdot}98 -.$$

EXERCISE 18.

Draw the shorter diagonal. We have then two triangles, in each of which we have two given sides, 483 and 397, and their included angle, 56° 14′. In each of these,

$$\text{As } 880 : 86 :: \tan 61^\circ 53' : \tan 10^\circ 22' - .$$
$$\angle A \doteq 72^\circ 15' - . \quad \angle C \doteq 51^\circ 31' + .$$
$$\text{As } \sin 72^\circ 15' : \sin 56^\circ 14' :: 483 : 421{\cdot}6 - .$$

Draw now the other diagonal. This also divides the parallelogram into two triangles, in each of which we have two given sides, 483 and 397, as before, the included angle $= 180^\circ - 56^\circ 14'$, and consequently its sine $=$ sin 56° 14′. In each of these,

$$\text{As } 880 : 86 :: \tan 28^\circ 7' : \tan 2^\circ 59' + .$$
$$\angle A \doteq 31^\circ 6' + . \quad \angle B \doteq 25^\circ 8' - .$$
$$\text{As } \sin 31^\circ 6' : \sin 56^\circ 14' :: 483 : 777{\cdot}3 + .$$

EXERCISE N.

Proceeding as in Exercise 18, we say, in order to find the shorter diagonal,

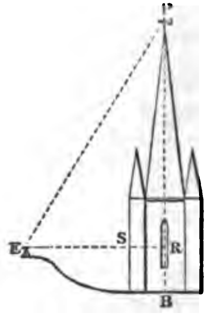
$$\text{As } 948 : 76 :: \tan 46^\circ 37\tfrac{1}{2}' : \tan 4^\circ 51' - .$$
$$\angle A \doteq 51^\circ 28\tfrac{1}{2}' - . \quad \angle C \doteq 41^\circ 46\tfrac{1}{2}' + .$$
$$\text{As } \sin 51^\circ 28\tfrac{1}{2}' : \sin 86^\circ 45' :: 512 : 653{\cdot}4.$$

To find the longer diagonal,

$$\text{As } 948 : 76 :: \tan 43^\circ 22\tfrac{1}{2}' : \tan 4^\circ 21' - .$$
$$\angle A \doteq 47^\circ 43\tfrac{1}{2}' - . \quad \angle C \doteq 39^\circ 1\tfrac{1}{2}' + .$$
$$\text{As } \sin 47^\circ 43\tfrac{1}{2}' : \sin 93^\circ 15' :: 512 : 690{\cdot}85.$$

EXERCISE 19.

$$
\begin{aligned}
ES &= 60{\cdot}0 \text{ feet.}\\
SR &= 17{\cdot}5\\
\hline
ER &= 77{\cdot}5\\
\therefore RP &\doteq 117{\cdot}4 +\\
RB &= 23{\cdot}0\\
\hline
PB &= 140{\cdot}4 + .
\end{aligned}
$$



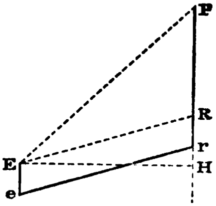
EXERCISE O.

$RP \doteq 79{\cdot}88 -$ feet.

EXERCISE 20.

Ee being the quadrant, and Pr the wall,

$$\begin{aligned} \angle PEH &= 41^\circ\ 15' \\ \angle REH &= 18\ \ 52 \\ \therefore \angle PER &\doteq 22\ \ 23, \\ \text{and } \angle P &= 48\ \ 45. \\ ER = er &\doteq 384. \end{aligned}$$



Then, in $\triangle EPR$, $\sin P : \sin E :: ER : PR$.
$\therefore PR \doteq 194{\cdot}5 -$, and $Pr = PR + Rr = 199{\cdot}5 -$.

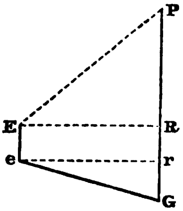
EXERCISE P.

As $\sin 61^\circ\ 22' : \sin 16^\circ\ 26' :: 496 : PR = 159{\cdot}9 -$.

EXERCISE 21.

Let PG be the corner of the wall. Then, in ⊾ erG, since $eG \doteq 90$ feet, and $rG = 21$ feet, we find er (or rather log er) by Rule 2 of Problem VII., Chapter VII. : thus,—

$90+21 = 111$	2·045323
$90-21 = \ \ 69$	1·838849
	2)3·884172
$ER = er$	1·942086
Tan ($PER = 39^\circ\ 27'$)	9·915332
	11·857418
Radius	10·000000
$PR = 72{\cdot}01 +$	1·857418.
$RG = 26{\cdot}00$	
$PG = 98{\cdot}01 +$.	



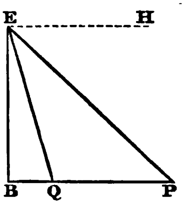
EXERCISE Q.

$ER \doteq 99. \quad PR \doteq 68{\cdot}68 -$.

EXERCISE 22.

In ⊿EBP, $\angle P \doteq \angle HEP = 9^\circ\ 28'$.

As rad. : cosec P :: EB : EP.



Cosec 9° 28′ ....10·783903
EB = 254 .... 2·404834

Radius ....10·000000

EP = 1544 + ... 3·188737

$\angle PEQ \doteq 72^\circ\ 40' - 9^\circ\ 28' = 63^\circ\ 12'$.

In △EQP, As sin Q : sin E :: EP : PQ.

Sin (E = 63° 12′)...............9·950650
EP ....3·188737

Sin (Q = 72° 40′)...............9·979816

PQ = 1444 + ...3·159571.

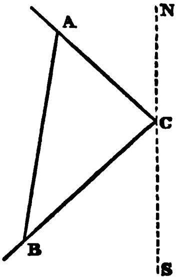
NOTE. This question might have been resolved by right-angled triangles alone, at three operations.

EXERCISE R.

As rad. : cosec 8° 14′ :: 168 : EP = 1173 +.
Sin Q = sin 78° 22′ : sin (E = 70° 8′) :: EP : PQ = 1126 +.

EXERCISE 23.

Let C be the corner, NS a meridian line through C, A the druggist's door, and B the surgeon's. Then,



$\angle ACN \doteq 51^\circ$
$\angle BCS = 48^\circ$

Sum of ∠s A and B = 99°
$\angle ACB = 180^\circ - 99^\circ = 81^\circ$.

Then, proceeding as in Problem VI. of Chapter VIII., we find,

$\angle A = 57^\circ\ 54'$, and
AB = 473·4 −.

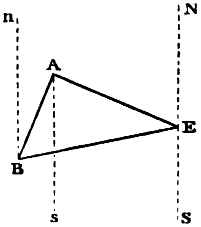
EXERCISE S.

$$\text{As } 500 : 100 :: \tan 65° : \tan 23° 13' - .$$
$$\text{Sin } (A = 41° 47') : \sin (C = 50°) :: 200 : 229{\cdot}9 + .$$

EXERCISE 24.

E being the vessel, and the dotted lines being meridians through A, B, and E, we have from Table V.,

$$\begin{array}{lrr} \angle EAs = \angle AEN = & 67° & 30' \\ \angle BAs = \angle ABn = & 22 & 30 \\ \hline \therefore \angle BAE \ldots\ldots\ldots\ldots \doteq & 90 & 0 \\ \hline \angle EBn = \angle BES \doteq & 78 & 45 \\ \angle ABn \ldots\ldots\ldots\ldots = & 22 & 30 \\ \hline \therefore \angle ABE \ldots\ldots\ldots\ldots \doteq & 56 & 15. \end{array}$$



Then, in ⊾ BAE,

$$\text{As radius} : \tan ABE :: BA : AE = 965{\cdot}3 + \text{, and}$$
$$\text{As radius} : \sec ABE :: BA : BE = 1161 - .$$

EXERCISE T.

In $\triangle$ BAE,

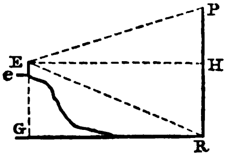
$$\text{As } \sin (E = 33° 45') : \sin (B = 45°) :: AB = 500 : AE.$$
$$\text{As } \sin (E = 33° 45') : \sin (A = 78° 45') :: 500 : BE.$$

EXERCISE 27.

First Method.

PR being the castle wall, E, the point of observation, and EH and GR, horizontal lines,

$$\begin{array}{lrr} \angle HEG \doteq & 90° & 0', \\ \angle HER = & 13 & 25. \\ \hline \therefore \angle GER \doteq & 76 & 35. \end{array}$$



Then, in ◺ EGR,

$$\text{As radius : tan } (E = 76^\circ\ 35') :: EG = 55 : GR.$$
$$\text{Log } GR^* = \log EH \doteq 2{\cdot}362800.$$

Again, in ◺ EHP,

$$\text{As radius : tan } (E = 10^\circ\ 41') :: EH : HP = 43{\cdot}50.$$
$$HR = EG \doteq 55{\cdot}00.$$
$$\overline{PR \doteq 98{\cdot}50.}$$

Second Method.

$90^\circ\ 0'$	$\angle PEH = 10^\circ\ 41'$
$\angle PEH = 10\ \ 41$	$\angle REH = 13\ \ 25$
$\angle EPH \doteq 79\ \ 19.$	$\angle PER \doteq 24\ \ \ 6.$

In ◺ EGR,

$$\text{As radius : sec } (E = 76^\circ\ 35') :: EG = 55 : ER.$$
$$\text{Log } ER^* \doteq 2{\cdot}374817.$$

In $\triangle$ PER,

$$\text{As sin } (P = 79^\circ\ 19') : \text{sin } (E = 24^\circ\ 6') :: ER : PR.$$

Third Method.

$$\text{As tan } 13^\circ\ 25' : \text{tan } 10^{c}\ 41' :: HR : PH = 43{\cdot}50 - .$$

EXERCISE U.

By the Third Method.

$$\text{As tan } 26^\circ\ 52' : \text{tan } 2^\circ\ 49' :: 113 : 10{\cdot}97 + .$$
$$PR \doteq 113 - (10{\cdot}97 +) = 102{\cdot}03 - .$$

EXERCISE 29.

We first find log BC = 1·071838,
Then log CD = 1·040045,
And, lastly, log DE = 1·078464.

---

* See Note to Exercise 29.

Note. We do not find the lines BC and CD themselves, but merely their logarithms. In fact, we might find the logarithm of DE at once, by adding together the logarithms of AB, sin BAC, sin CBD, and sin DCE, and from the sum subtracting the sum of the logarithms of sin ACB, sin CDB, and sin CED. The more circuitous method, however, is the better for learners.

EXERCISE V.

$$\text{Log BC} \doteq 3{\cdot}806192. \quad \text{Log CD} \doteq 3{\cdot}852476.$$
$$\text{Log DE} \doteq 3{\cdot}815300.$$

EXERCISE 30.

$$\angle EPF \doteq \angle PFR \underset{\cdot}{-} \angle E = 5^\circ\ 23'.$$

$$\text{In } \triangle EPF,\ \sin P : \sin E :: EF : PF = 806{\cdot}8 + .$$
$$\text{Log PF} \doteq 2{\cdot}906783 - .$$

This logarithm is retained for the next operation.

$$\text{In } ◺\, PRF,\ \text{radius} : \sin F :: PF : PR = 356{\cdot}2 + .$$

EXERCISE W.

$$\angle EPF \doteq 6^\circ\ 23'.$$
$$\text{Log FP} \doteq 2{\cdot}858449. \quad \text{Log PR} = 2{\cdot}430144.$$

EXERCISE 31.

Reduce the chains to feet, and then find the height of the hill, as in the preceding exercise. At one operation the whole will stand thus:

660 feet........................	2·819544
Sin (E = 6° 25′)..............	9·048279
Sin (F = 9° 14′)..............	9·205354
Radius..........................	10·000000
Sin (EPF = 2° 49′)...........	8·691438
PR = 240·8 +.................	2·381739

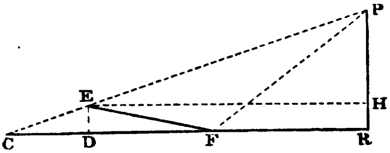
EXERCISE X.

$$\angle \mathrm{EPF} \doteq 1^\circ\ 56'.$$

$$\text{Log FP} \doteq 3{\cdot}253055. \quad \text{Log PR} \doteq 2{\cdot}293968.$$

Or it may be performed at one operation, in the same manner as Exercise 31.

EXERCISE 32.



$\angle \mathrm{PFR} \doteq 9^\circ\ 26'$;
$\angle \mathrm{PCR} = 5\ \ 59.$
$\angle \mathrm{CPF} \doteq 3\ \ 27.$

$\angle \mathrm{ECF} = \angle \mathrm{PEH} \doteq 5\ \ 59.$
$\angle \mathrm{EFC} \ldots\ldots\ldots\ldots \doteq 3\ \ 4.$
$\angle \mathrm{PEF} \ldots\ldots\ldots\ldots \doteq 9\ \ 3.$

In $\triangle$ PEF, Sin P : sin E :: EF = 660 : PF.
(Log PF $\doteq 3{\cdot}236829$.)

In ⊿ PRF, radius : sin F :: PF : PR = 282·8 –.
In ⊿ EDF, radius : sin F :: EF : ED = 35·3 +.
PH $\doteq$ 247·5 –.

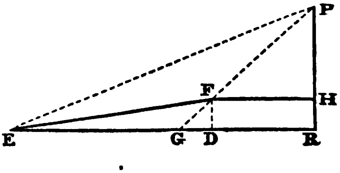
EXERCISE Y.

$$\angle \mathrm{CPF} \doteq 2^\circ\ 27'. \quad \angle \mathrm{PEF} \doteq 5^\circ\ 19'.$$

Log FP $\doteq 3{\cdot}155526$. PH $\doteq$ 211·5 –. RH $\doteq$ 8·4 +.

EXERCISE 33.

$\angle \mathrm{PGR} = \angle \mathrm{PFH} \doteq 7^\circ\ 40'.$
$\angle \mathrm{PEG} \ldots\ldots\ldots\ldots \doteq 4\ \ 23.$
$\angle \mathrm{EPG}$ or $\angle \mathrm{EPF} \doteq 3\ \ 17.$



In ◺ EDF,

As EF : FD :: radius : sin FED.

$$\therefore \angle FED \doteq 1^\circ\ 14' -.$$
$$\angle PED \doteq 4\ \ 23$$
$$\angle PEF \doteq \underline{3\ \ \ 9} +.$$

In $\triangle$ EPF, sin P : sin E :: EF : PF.
(Log PF $\doteq$ 2·880739.)

In ◺ PHF, radius : sin F :: PF : PH $= 101+$
$17 + 93 = 110$
Answer, $\underline{211}$ + feet.

The omission of the seconds from the angle PEF does not affect the answer as far as the figures are carried.

EXERCISE Z.

(See the Diagram of Exercise 32.)

$$\angle FEH = \angle EFD \doteq 1^\circ\ 52' -.$$

In $\triangle$ EPF,
Sin ($P = 3^\circ\ 39\frac{1}{2}'$) : sin ($E = 5^\circ\ 42'$) :: EF $= 800$: FP.
$\therefore$ Log FP $\doteq$ 2·192172.

In ◺ PRF,
As radius : sin ($F = 7^\circ\ 29'$) :: FP : PR $= 162·2 -$.

EXERCISE 34.

In $\triangle$ EPF,
Sin ($P = 30^\circ$) : sin F $=$ sin 70° :: EF $= 80$: EP $= 150·35 +$.

In $\triangle$ PER,

EP + ER : EP − ER :: tan $\frac{1}{2}$ (R + P) : tan $\frac{1}{2}$ (R − P.)

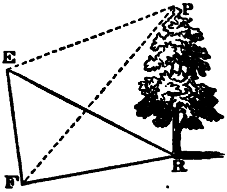
I. e. As 250·35 : 50·35 :: tan 70° : tan 28° 55′ +.
$\therefore \angle EPR \doteq 41^\circ\ 5' -$.

In $\triangle$ PER, again, sin P : sin E :: ER : PR $= 97·81 +$.
$\therefore$ Pr. $\doteq$ 102·81 +.

EXERCISE AA.

In △EPF, sin 16° 28′ : sin 21° 42′ : : 66 : FP = 86·09 + .
In △PFR, 152·09 : 20·09 : : tan 70° 55′ : tan 20° 54′ − .
In △PFR, sin 50° 1′ : sin 38° 10′ : : 66 : PR = 53·23 − .

EXERCISE 35.



In △EPF,

As sin P : sin F : : EF : EP.
Log EP ≐ 2·476881.

In ◺ERP,

As rad : sin E : : EP : PR = 112·8.

EXERCISE BB.

In △ERF, sin 21° 46′ : sin 79° 56′ : : EF = 132 : ER.
Log ER ≐ 2·544664.

In ◺ERP, rad : (tan E = 19° 46′) : : ER : PR = 126·0 − .

EXERCISE 36.

In △ERF,

As sin (R = 2° 47′) : sin (F = 87° 33′) : : EF = 2244 : ER.

Log (ER = 46170 feet)........4·664356
5280 feet in a mile............3·722634
ER = 8·744 + miles...........0·941722.

In ◺ERP,

As radius : tan (E = 3° 18′) : : ER : PR = 2662 + .

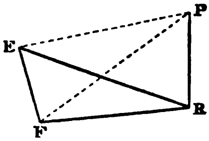
EXERCISE CC.

In △ERF, sin 2° 47′ : sin 91° 13′ : : 1980 : ER = 40770 − .
Log ER ≐ 4·610295.

In ◺ERF, rad : tan 2° 53′ : : ER : PR = 2053 + .

EXERCISE 38.

In △ERF, sin R : sin F :: EF : ER.
In ◺ERP, radius : sec E :: ER : EP.
In ◺ERP, radius : tan E :: ER : PR.



Sin (F = 86° 21′)...9·999118
EF = 25·27 chains 1·402605
66 ft. in a chain....1·819544
Sin (R = 4° 24′)....8·884903
ER, in feet..........4·336364
Sec (E = 1° 48′)...10·000214
Radius..............10·000000
5280 ft. in a mile...3·722634
EP = 4·111 − miles 0·613944

ER, in feet..........4·336364
Tan (E = 1° 48′)...8·497293
Radius.............10·000000
PR = 681·8 ft......2·833657

EXERCISE DD.

As sin 4° 22′ : sin 84° 2′ :: 12·88 : ER.
Log ER, in feet, ≐ 4·045494.
As radius : sec 2° 12′ :: ER : EP = 2·105 − miles.
As radius : tan 2° 12′ :: ER : PR = 428·2 + feet.

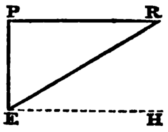
To the height computed 5 feet must be added for the height of the instrument, both in this and in the preceding exercise.

EXERCISE 45.

60 min. : 8 min. :: 15 miles : 2 miles.
∴ PR ≐ 2; and ∠PRE ≐ ∠REH.

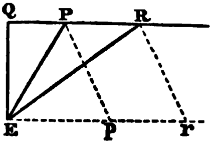
In ◺EPR,

Rad : tan R :: PR : EP, or
Rad : tan 31° :: 2 : 1·202 −.



EXERCISE 47.

Let Q, P, and R be the three successive positions of the cloud. Then, since the sun's rays may be regarded as parallel over the small variation of time and place implied in the question, and since the motion of the cloud is parallel to the surface of the ground, the distance PR may be regarded as equal to that passed over by the shadow on the plain,—viz. pr, or 923 yards. Then,



In △ EPR,

As $\sin(E = 23^\circ) : \sin(R = 42^\circ) :: PR = 923 : EP$.

$\text{Log } EP \doteq 3{\cdot}198835$.

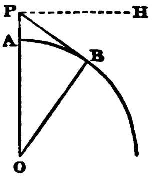
In ◺ EQP,

$\text{Rad} : \sin(P = 65^\circ) :: EP : EQ = 1433 - \text{yds.}$

$= 0{\cdot}814 - \text{mile.}$

EXERCISE 49.

In ◺ PBO,



$OB \doteq 3956$, and $OP = 3959$.

As $OB : OP :: \text{rad} : \sec O$,

or $OP : OB :: \text{rad} : \cos O$.

$\therefore \angle O \doteq 2^\circ\ 14' = \angle HPB$.

EXERCISE 56.

In △PQR, by Pr. VII. of Ch. VIII.,

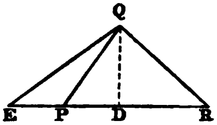
$6 : 5 + 4 :: 5 - 4 : 1{\cdot}5 = DR - DP$.

$\therefore DP \doteq 2{\cdot}25$.

In ◺ PDQ,

$DP : PQ :: \text{rad} : \sec P$.

$\therefore \angle P \doteq 55^\circ\ 46' +$.



In $\triangle$ EQP,

Sin (E = 36°) : sin (P = 55° 46′) :: PQ = 4 : EQ = 5·63 −.
And, sin E : sin (Q = 19° 46′) :: PQ : EP = 2·30 +.

EXERCISE HH.

DP $\doteq$ 1·375. $\angle$DPQ $\doteq$ 46° 34′ +.
As sin 45° : sin 46° 34′ :: 2 : EQ = 2·054 −.
As sin 45° : sin 1° 34′ :: 2 : EP = ·07733 −.

Or, if we choose, after finding DR = 2·625, and DP = 1·375, we may find QD = 1·452 + = ED, and EQ = $\sqrt{(ED^2 + DQ^2)} = \sqrt{(2ED^2)} = ED \times \sqrt{2}$ = 2·054 −. Then EP $\doteq$ ED − PD = (1·452 +) − 1·375 = ·077 +.

FINIS.

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